**1. Number System**

**Solution Exercise – Easy**

1. (a) : We see that 16 = 11 + 5, 9 = 2 + 7, 5 = 2 + 3, 12 = 5 + 7

So, 16, 9 and 5 can all be written as the sum of two prime numbers.

But 23 cannot be expressed in the same way.

2. (d) : Any prime number greater than 5 can be expressed in the form 6*k* ± 1, where *k* is a natural number.

(6*k* ± 1)2 = 36*k* ± 12*k* ± 1

= 12*k* (3*k* ± 1) + 1

For all the values of *k*, *k* (3*k* ± 1) is always divisible by 2.

Hence, when (*P*2 + 1) is divided by 24, the remainder is 1.

3. (c) : *p* = 6*N* ± 1

So, *p*2 + 17 = (6*N* ± 1)2 + 17

= 36*N*2 ± 12*N* + 1 + 17

= 12(3*N*2 ± *N*) + 18

18 when divided by 12, the remainder is 6.

4. (b) : Let the prime number are: *a, b, c* and *d*

It is given that a × b × c = 385 .....(i)

And *b × c × d* = 1001 .....(ii)

Now divide equation (ii) by equation (i)

= 

⇒ 

So, largest number = 13

**Alternate** **Method:**

Since 385 is divisible by 5 and not divisible by 3.

So, the four numbers are 5, 7, 11 and 13 and hence the largest prime number is 13.

5. (c) : If *n*3 is odd, then *n* and *n*2 will also be odd. It can be checked for any odd integer.

If *n* = 3, *n*2 = 9, *n*3 = 27

6. (a) : Statement **I:**

As *x* and *y* are both odd

So, *x + y* is even.

Hence, (*x + y*) *x* is even.

So, choice (I) is false.

Statement **II:**

*x + z* is odd as *z* is even.

So, choice (II) is true.

Statement **III:**

*x + z* is odd as *x* is odd, and *x*3 is odd.

Hence, (*x + z*) *x*3 is odd.

So, choice (III) is true.

Statement **IV:**

As *x* is odd and *z* is even.

*x + z* is odd

Hence, *x*(*x + z*)3 is odd.

So, choice (IV) is true.

7. (c) : Let the number be of the form 10*x + y*. Then its reverse be 10*y + x*

∴ 10*x + y* + 10*y + x* = 99 or, 11*x* + 11*y* = 99 or,

*x**+ y* = 9

The sum of the digits of original number is 9.

8. (b) : Let the number be *ab*.

*b – a* = 2 .....(i)

(*a + b*)(10*a + b*) = 280

⇒ (*a + a* + 2)(10*a + a* + 2) = 280

[From (i) *b* = *a* + 2]

⇒ 2(1 + *a*)(11*a* + 2) = 280

⇒ (1 + *a*)(11*a* + 2) = 140

⇒ 11*a* + 2 + 11*a*2 + 2*a* = 140

⇒ 11*a*2 + 13*a* – 138 = 0

⇒ 11*a*2 + 46*a* – 33*a* – 138 = 0

⇒ *a*(11*a* + 46) – 3 (11*a* + 46) = 0

⇒ (*a* – 3)(11*a* + 46) = 0

*a* = 3 or – .

But the value of *a* can’t be negative.

Hence *a* = 3 & *b* = 3 + 2 = 5

∴ a + b = 3 + 5 = 8.

9. (c) : Since he has covered twice the distance which yet he had to cover. It means he has covered of the whole journey and remaining journey is  of the whole.

10. (d) : Since *a* > 0 then at *a* = 1, we get *b* = 2 and for the rest values we get *b* > 2

Also for 0 < *a* < 1 ; *a* +  > 2

and for *a* > 1 ; *a* +  > 2

Hence, *a* +  = b ≥ 2

11. (a) : By observing 

Numerator is decreasing and denominator is same.

Now we have to compare  and.



∴  is the greatest.

12. (c) : Let the number be ‘*n*’

So,  = 39

 = 39

So, *n* = 40

13. (a) : 53*x* – 35*x* = 540

18*x* = 540 or *x* = 30

∴ Correct product = 53 × 30 = 1050

14. (c) : 112 = 121

(111)2 = 12321

(1111)2 = 1234321

........

(1111111111)2 = 12345678987654321

So, 

= 111111111

15. (c) : 302 = 900,

312 = 961 and 322 = 1024.

Hence, 1024 is the required answer.

16. (c) : (85)2 is the closest perfect square above 7147. Hence, 7225 – 7147 = 78 is the required answer.

So, 78 is the required answer.

17. (c) : 

= 

18. (b) : Let the first applicant is female. The remaining applicants = 5005 – 1 = 5004

For maximum female applicants, for every six applicants, there should be a female.

∴ Number of females = 1 +  = 1 + 834 = 835

19. (b) :  = 79 and (73)2 = 76. Hence, clearly 79 > 76.

20. (b) : (*a + b*)– 1 × (*a*– 1 + *b*– 1) = 

=  = (*ab*)– 1

21. (b) : 5*x* – 1 + 5*x* + 5*x* + 1 = 775

⇒ 5*x* (5 – 1 + 1 + 51) = 775

⇒ 5*x* = 775

⇒ 5*x* × = 775

⇒ 5*x* = 125 = 53

⇒ *x =* 3

Alternate **Method:**

Put values from the options and check.

22. (b) : 112 + 114 ÷ 113 – 11 + (0.5) ×112

= 112 + 11 – 11 + 0.5 ×112

= 121 + 11 – 11 + 60.5

= 121 + 0 + 60.5

= 181.5

23. (d) : *x* = + 1

⇒ *x* + 

= 

= 

**Alternate Method:**



= 

So, 

24. (c) : The value of *A* is negative.

*D* = 1.44, *C* = 0.09, *B* = 0.16,

So, *D* > *B* > *C* > *A*.

Alternate **Method:**

As we can see that the value of *A* is negative, so it should be in the extreme right in the given options, which can be only seen in option (c).

25. (b) : x = 6 – 

∴ 

= 6 + 

26. (d) : Given expression

= 

Alternate **Method:**

Put values of *n* to get the value of the expression.

For example taken = 0, 1 or – 1

27. (a) : 

= 

= (55 + 45) = 100

28. (b) : 

= 

= 

Square root = 

Alternatively we can use options to get the answer.

Square the values and check which comes equal to.

29. (c) : There are 20 4’s in the given 100 numbers.

30. (c) : The digit 6 appears 20 × 2 = 40 times in units and tens place and 100 times in the hundred place.

Total = 40 + 100 = 140 times

31. (a) : log12 27 = *a* or, 

or, *a* log 12 = log 33

or, *a* log (3 × 4) = 3 log 3

or, *a* [log 3 + log 4] = 3 log 3

or, *a* log 3 + *a* log 4 = 3 log 3

or, *a* log 22 = (3 − a) log 3

or, 2*a* log 2 = (3 − a) log 3

or,  ..... (1)

Now, log6 16 = 

= 

32. (b) : *ax* = *b* or, log*a* *b* = *x*

or, *by* = *c* or, log*b* *c* = *y*

or, *cz* = *a* or, log*c* *a* = *z*

∴ *x ×* *y* × *z* = log*a* *b* × log*b* *c* × log*c* *a* = 1

33. (d)

810 = (23)10 = 230

∴ required answer = [30 log10 2 + 1]

= [30 × 0.3010] + 1 = (9.03) + 1 = 9 + 1 = 10

34. (c) : Let log6 = *x*

then  = (6)*x* [ log*a* *x* = *y* ⇒ *x* = *ay* ]



⇒ 

⇒ 

35. (b) : log7 log5= 0

⇒ log5= (7)0 = 1

[ log*a* *x* = *y* ⇒ *x* = *ay* ]

⇒  = (5)1 = 5

= 

∴ *x* = 0

36. (a) : The sum of the digits of the number should not be divisible by 9. Option (a) is divisible by 21 but not 9.

37. (a) : 5*a*79 should be divisible by both 11 and 3.

So, for the divisibility of 3,

5 + *a* + 7 + 9 = Multiple of 3

21 + *a* = Multiple of 3. So, *a* = 0 or 3 or 6 or 9

For Multiple of 11.

*a* – 3 = 0

⇒ *a* = 3.

So, a = 3

38. (c) : Sum of all digits must be divisible by 9.

4 + 5 + 1 + \* + 6 + 0 + 3

= 19 + \* should be divisible by 9.

∴ \* = 8

39. (c) : *n* + (*n* + 2) + (*n +* 4) = 3*n* + 6 = 3(*n* + 2),

where *n* = 2, 4, 6, .....

If *n* = 2 ; Sum = 12

& *n* = 4 ; Sum = 18

So, the sum is always divisible by 6.

40. (a) : Since it is an odd number and it cannot be divisible by 4 & 6. The value is divisible by 3.

41. (d) : The number should be divisible by both 11 & 8 as 11 & 8 are coprime to each other

42. (a) : Put values of ‘*n*’ and check.

43. (d) : Any number which is formed by writing any digit 6 times is divisible by 1001.

999999 = 999 × 1000 + 999

= 999 × (1001)

As, 1001 = 7 × 11 × 13

44. (d) : 553 + 173 – 723

(If *a + b + c* = 0, the *a*3 + *b*3 + *c*3 = 3*abc*)

= – 3 × 55 × 17 × 72

So, it is divisible by 3 and 17.

45. (a) : *N* = *n*(*n* + 1)(*n* + 2)(*n* + 3)(*n* + 4)

at *n* = 1, *N* = 120 = 1 × 2 × 3 × 4 × 5

at *n* = 2, *N* = 720 = 2 × 3 × 4 × 5 × 6

at *n* = 3, *N* = 2520 = 3 × 4 × 5 × 6 × 7

at *n* = 4, *N* = 6720 = 4 × 5 × 6 × 7 × 8 etc.

Hence, the largest possible number is 120. (HCF of (12, 720,....))

46. (c) : *N* = 84*K* + 57

= 28 (3*K*) + 56 + 1

= 28 (3*K* + 2) + 1

47. (d) : *n* = 5*k* + 2

⇒ *n*2 = 25*k*2 + 10*k* + 4

*n*2 = 5*k* (5*k* + 2) + 4

Hence the remainder when n2 is divided by 5 is 4.

48. (a) : 2256 = (24)64 = (16)64 = (17 – 1)64 = 17*K* + (– 1)64

= 17*K* + 1

49. (c) : 9091 = (91 – 1)91 = 91 + (– 1)91

= 13 × 7 − 1

Hence, remainder is – 1 or 12.

50. (a) : 234 + 67 = 2 × (23)11 + 67

= 2 × (8)11 + 67

= 2 × (1)11 + 67

= 2 + 67 = 69

69 when divided by 7. The remainder is 6.

51. (c) : *an + bn* is always divisible by *a + b* when *n* is odd.

∴ 1523 + 2323 is always divisible by 15 + 23 = 38.

As 38 is a multiple of 19, so 1523 + 2323 is divisible by 19.

∴ We get a remainder of 0.

52. (c) : 15 when divided by 4, leaves a remainder of – 1.

– 116 = 1.

∴ 271 = 27

So, the unit digit is 7.

53. (d) : The respective units digits for the six parts of the expression would be the unit digit of the product of:

1 × 4 × 7 × 6 × 5 × 6 → required answer is 0.

54. (d) : Unit digit of the product will be the unit digit of the product 4 × 3 × 6 × 5 which is ‘0’.

55. (c) : Unit digit of 3153 is 3.

& unit digit of 7162 is 9.

So, required unit digit will be the unit digit of the product 3 × 9 *i.e*. 7.

56. (d) : Unit digit of 3200 is 1.

Unit digit of 4500 in 6.

So, the required unit digit will be the unit digit of the product 1 × 6 *i.e*. 6.

57. (d) : 411 + 412 + 413 + 414 + 415

= 411 (1 + 41 + 42 + 43 + 44) = 411 × 341.

The factors of 341 are: 1, 11, 31 and 341. Thus, we can say that the values in each of the three options would divide the expression.

***Solutions for 58 and 59:***

The given condition says that Toffee < Chocolate < A packet of chips .

Also, since the least cost of the three is Rs. 12, if we allocate a minimum of 12 to each, we use up 36 out of 41. The remaining 5 can be distributed as 0, 1, 4 or 0, 2 and 3 giving possible values of

Case **I:**  12, 13 and 16 or

Case **II:** 12, 14 and 15.

58. (a) : In both the cases, the cost of the toffee is 12.

59. (c) : If the cost of the a packet of chips is not divisible by 4, it means that Case II holds true. For this case, the cost of the chocolate is 14.

***Solutions for* 60 and 61:**

60. (b) : Since both *A* and *B* are prime numbers, the number of factors would be (1 + 1) (1 + 1) = 4.

61. (c) : Since both *A* and *B* are prime numbers, the number of factors would be (2 + 1) (1 + 1) = 6.

62. (d) : Since both *A* and *B* are prime numbers, the number of factors would be (3 + 1) (2 + 1) = 12.

63. (d)

*AB* = 64

(*A, B*) may be any of the followings

|  |  |
| --- | --- |
| **Case** | **Sum** |
| 1 × 64 | 65 (1 + 64) |
| 2 × 32 | 34 (2 + 32) |
| 4 × 16 | 20 (4 + 16) |
| 8 × 8 | 16 (8 + 8) |

Hence, *A + B* cannot be 35.

64. (d) : Number of factors of 48 = 10

Total number of ways =  = 5

65. (c) : HCF of 32, 36, 72 and 24 = 4

LCM of 32, 36, 72 and 24 = 288

∴ 

66. (d) : For the GCD take the least powers of all common prime factors.

Thus, the required answer would be 23 × 3.

67. (d) : The required number is the L.C.M. of (4, 6, 7) + 2

= 84 + 2 = 86

68. (a) : The difference between the divisor and the remainder is same.

32 – 10 = 22

40 – 18 = 22

72 – 50 = 22

Now the L.C.M of 32, 40, 72 is 1440

∴ The least number of flowers is 1440 – 22 = 1418.

69. (a) : Let the numbers be 3*x* and 4*x*

3*x* × 4*x* = 10800

⇒ 12*x*2 = 10800 ⇒ *x* = 30

∴ 3*x* = 90 and 4*x* = 120.

**Alternate Method:**

Use options to get the requested numbers.

70. (c) : LCM of 2, 3, 4, 5 and 6 is 60 .

The numbers 61, 121, 181 etc would give us a remainder 1 when divided by 2, 3, 4, 5 and 6.

The least 3 digit number in this series is 121.

71. (c) : As the intervals of time are 12, 18 and 36 min.

LCM of 12, 18 and 36 is 36.

So, the bells will ring together after 36 minutes .

So, next time they will ring together is 12:01 P.M.

72. (d) : If the cookies are distributed among 40 kids then 4 are left then the number of cookies is 40 *K*1 + 4, where *K*1 is a natural number.

Now, 40 *K*1 + 4 = 41 *K*2 + 8

40 *K*1 = 40 *K*2 + *K*2 + 4

*K*1 = *K*2 + 

The minimum value of *k* for this is 36.

Hence, the minimum number of cookies

= 41 × 36 + 4 = 1480.

73. (d) : The greatest length which can divide each of the given length will be the HCF of the given sides which is 9.

74. (b) : The HCF of 576 and 448 is 32. Hence, each section should have 32 children. The number of sections would be given by:

=  = 18 + 14 = 32

75. (b) : The HCF of P, Q, R and S would be the HCF of 26 and 39, *i.e.* 13.

76. (a) : The number of zeros would be given by the highest power of 5:

= 

= 218 + 43 + 8 + 1 = 270

77. (c) : *N*! is having 37 zeros at its end, so *N* = 150 (obtained by Hit and Trial as *N* is approximately four times the number of zeros).

Obviously, 150 ≤ *N* ≤ 154 is the answer.

So, five values are possible.

78. (d) : Power of 5 =  = 10 + 2 = 12.

So, value of *p* = 12

79. (b) : The power of 20 which would divide 155! would be given by the power of 5’s which would divide 155! since 20 = 22 × 5 and the number of 22 in any factorial would always be greater than the number of 5’s in the factorial.

Power of 5 = 

= 31 + 6 + 1 = 38.

80. (b) : 1 to 1600 A.D. = 0 odd days

1601 A.D. to 1700 A.D. = 5 odd days

1700 A.D. to 1762 A.D. = 0 odd days

1 Jan 1763 to 15 Feb 1763 = 4 odd days

Total odd days = 9 = 2 odd days.

So, it was a Tuesday.

81. (c) : 1 to 1600 A.D. = 0 odd days

1601 A.D. to 1900 A.D. = 1 odd days

1901 A.D. to 1978 A.D. = 6 odd days

1 Jan 1979 to 3 Oct 1979 = 3 odd day

Total odd days = 10 = 3 odd days

So, it was a Wednesday.

82. (b) : For each normal year if we add 1 and for each leap year we add 2. We will get 11th October 1985 as Monday given that 11th October 1962 was a Monday.

83. (b) : 1st June 1857 – 31st Dec 1857 = 4 odd days

1858 – 1918 = 5 odd days

1 Jan 1919 – 31 Aug 1919 = 5

Total odd days = 14 = 0 odd day = Saturday (as the reference is Sunday)

84. (b) : Minimum number of days in 7 consecutive years

= 365 × 7 = 2555 days.

(e.g. 1797, 1798, 1799, 1800, 1801, 1802, 1803 are all non-leap)

85. (d) : (32312)4 = (1110110110)2 = (3B6)16.

86. (c) :

|  |  |
| --- | --- |
| 12 | 1982 |
| 12 | 165 – 2 |
| 12 | 13 – 9 |
|  | 1 – 1 |

The required number is 1192.

87. (a) : (25)*b* = 2*b* + 5 & (52)*b* = 5*b* + 2

According to the question:

2 + 5*b* = 2(5 + 2*b*)

⇒ 2 + 5*b* = 10 + 1*b*

⇒ *b* = 8

**Solution Exercise – Medium**

1. (b) : The three consecutive odd integers are – 3, – 1 and 1. Product = – 3 × – 1 × 1 = 3

2. (c) : *x* can assume only one value, which is 2.

At *x* = 2, *x*2 + 3 = 7, which is also prime.

Again at *x* = 3, 5, 7, 11, ...

*x*2 + 3 = an even number which can not be a prime number.

3. (a) : Let the number be ab

So (10b + a) – (10a + b) = 27

9b – 9a = 27

b – a = 3

So, the number except 14 are 25, 36, 47, 58 and 69 i.e. 5 numbers.

4. (b) : Using options we find the only option (b) satisfies the given condition

= (5 + 4)2 – 54 = 27

5. (c) :

|  |  |
| --- | --- |
| **Range** | **Numbers** |
| 287 − 299 | 13 |
| 300 − 799 | 95 (19 × 5) |
| 800 − 803 | 1 |
| Total | 109 |

6. (c) : The four numbers satisfying the condition are 164, 364, 649 & 816

7. (b) : (*BE*)2 = *MPB*

The value of *B* should be 1 as only then the unit digit will be same.

Now 112 = 121

This case is discarded as the values of *M* & *B* will be same (and not distined)

So, 192 = 361 is the only possible case

So *M* = 3

8. (d) : Sum of first 10 odd numbers = 100

Sum of next set of 10 odd numbers = 120

Sum of next set of 10 odd numbers = 140

.

.

.

.

Sum if largest 10 odd numbers from the set = 1900

Possible values are 100, 120, 140, ........... 1900

Number of possible values = 901



9. (c) : We have to find an approximate value of n such that the sum of numbers from 1 to *n* is just more than 177.

Sum of numbers from 1 to 19 = 190

So the boy must have missed 190 - 177 *i.e*. 13.

Note: No other case will be possible as the difference will be more than ‘n’

10. (d) : The sum of first ten natural number is 55

Now, the sum of next set of ten natural numbers is 65 (from 2 to 11)

Now, the sum of next set of ten natural numbers is 75 (from 3 to 12)

So, possible values are 55, 65, 75, 85, ........

All values with unit digit 5 are possible starting from 55.

11. (a) : 1 – 3 + 5 – 7 + 9 – 11 + ...... + 197 – 199

= (– 2) + (– 2) + (– 2) = ..... + (– 2) [50 times]

= 50 × (– 2) = – 100

12. (d) : Go through options

1 + 2 + 3 + ...... + *k* = *N*2 (a perfect square)

=  = *N*2

=  = (1)2 → a perfect square number,

=  = 4 × 9 = (2 × 3)2 → a perfect square number.

and  = 49 × 25 = (7 × 5)2 → a perfect square number.

13. (a) : 12 – 22 + 32 – 42 + 52 – 62 + .... + 1972 – 1982 + 1992

= – [22 – 12 + 42 – 32 + 62 – 52 + ...... 1982 – 1972] + 1992

= – [(2 + 1) (2 –1) + (4 + 3) (4 – 3) + (6 + 5) (6 – 5) + ... (198 + 197) (198 – 197)] + 1992

= – [2 + 1 + 4 + 3 + 6 + 5 + .... 198 + 197)] + 1992

= – [1 + 2 + 3 + 4 + 5 + 6 + .... 197 + 198)] + 1992

= –  + 1992

= – 99 × 199 + 1992 = 199 (199 – 99)

= 199 × 100 = 19900

14. (a) : Given *tn* = , *n* = 1, 2, ....

Therefore, *t*3 = , *t*4 = , *t*5 = , ......, *t*53 =

Therefore, *t*3 × *t*4 × *t*5 × ..... × *t*53

= 

= 

15. (d) : 720 = 8 × 9 × 10

The given numbers are 8, 9, 10

Therefore, Sum = 8 + 9 + 10 = 27

16. (d) : The numbers are in an arithmetic progression. From the statements we can find that 8.5, 10 and 11.5 can be the three values. And hence the middle value is 10.

17. (d) :  should be a perfect square. The first value of *n* when this occurs would be for *n* = 8. Thus, on the 8th of January the required condition would come true.

18. (a) : There are two brothers and two sisters.

So, total number of brothers and sisters = 4

19. (c) :

|  |  |  |
| --- | --- | --- |
|  | Laxman | Ram |
| Fired shots | 5 | 7 |
| Hit shots | 2 | 3 |
| Missed shots | 3 | 4 |

When Ram missed 32 shots, it means Laxman missed 24 shots.

When Ram missed 24 shots, it means Laxman hit 16 shots.

20. (c) : *A*2 + *A*3 = *A*2 (*A* + 1)

For *A*2 + *A*3 be a perfect square, (*A* + 1) should be a perfect square.

And we know there are 10 perfect square till 100.

But we cannot take *A* + 1 = 1 ⇒ *A* = 0

So there are 9 numbers for which *A*2(*A* + 1) will be a perfect square.

21. (a) : The only possible number is 729 which is the cube of 9 and square of 27.

So, 7 × 2 × 9 = 126

22. (b) : For being a perfect square, the last digit of the number should be 1, 4, 5, 6 and 9.

And the digital sum of the number should be:

1, 4, 9 and 7.

23. (a) : Let capacity = *N*

⇒ 

⇒ 

⇒ 

*N* = 90

Alternate **Method:**

It can be solved by options:

Let’s consider option (a), then

90 ×  = 75

75 – 5 + 2 = 72

Again 90 ×  = 72

Hence, this option is correct.

24. (a) : Let the four integers be *x*, *x* + 1, *x* + 2 & *x* + 3

So *N* = 1 + *x*(*x* + 1)(*x* + 2) (*x* + 3)

= 1 + *x* (*x +* 3)(*x* + 1)(*x* + 2)

= 1 + (*x*2 + 3 *x*)(*x*2 + 3*x* + 2)

Now *x*2 + 3*x* = *K*

So *N* = 1 + *K* (*K* + 2)

= 1 + *K*2 + 2*K*

= (*K* + 1)2

So, *N* is a perfect square.

Also, *N* will odd as the product of any four consecutive integers be even.

The product will also be a multiple of 3 where as *N* when divided by 3 the remainder will be 1.

25. (c) : 3*m* = 9*n* ⇒ *m* = 2*n*

and 4(*m + n*) or3m =32*n* = 16*mn =* 42*mn*

⇒ (*m + n* + 2)*n* = 2*mn*

⇒ *m + n* + 2 = 2*m*

⇒ *m – n* = 2

⇒ *n* = 2 & *m* = 4 (∴ *m* = 2*n*)

∴ *m* = 4

26. (c) : For *A* to be the least possible number *z* < *y* < *x.*

But *z* *≠* 1, since *x* *≠ y ≠ z.*

So, the least possible value of *z* is 2.

then, *z*8 = 28 = 256 = 44 = 162

So, A = 256

27. (b) : LCM of 2, 3, 4, 6, 12 = 12









So, 

28. (a) : 33322 and 33332

⇒ (34)830 × 32 and 33332

Thus (81)830 × 32 > 33332

Again 33322 and 33322

⇒ (36)553 × 34 and (333)22

Thus (729)553 × 34 > (33)22

Again 33322 and 22333

⇒ (33)1107 × 3 and 22333

Thus (27)1107 × 3 > 22333

29. (c) : 1 + 2 + 22 + .... + 231 = 232 – 1

Hence, the average will be:

, which lies between 226 and 227.

30. (d) :  = [True] (option (a))

= 

=  [True] (option (b) & (c))

= 

=  [False]

31. (b) : In number from 1 to 399 there are 19 × 4 = 76 numbers and from 400 to 500 there are 100 numbers in which the digit 4 comes at any place.

So, in total there are 176 numbers.

32. (a) : In numbers from 300 to 399 there are 18 numbers and from 400 to 500 there are 81 numbers in which the digit 4 comes only once.

33. (d) : Given log*b* *a* = 3 and log*b* 8*a* = 4



⇒ loga 8*a* =  ⇒ log*a* 8 + log*a* *a* = 

⇒ log*a* 8 = 

⇒ 8 = *a*1/3

⇒ *a* = 512

34. (b) : log 15 = log (3 × 5)

log 3 + log 5 = *p* ..... (1)

log 20 = log (22 × 5)

2 log 2 + log 5 = *q* ..... (2)

Also, log10 = log 2 + log 5 = 1 ..... (3)

We can express log 2 and log 3 in terms of *p* and *q* as follows

(2) ⇒ 2 log 2 + 1 − log 2 = *q* ⇒ log2 = *q* − 1

(3) ⇒ log 5 = 2 − *q* and

(1) ⇒ log 3 = *p* + *q* − 2

Now log 12 = 2 log 2 + log 3

= 2 (*q* − 1) + (*p* + *q* − 2) = *p* + 3*q* − 4

35. (b) : log(*a* − 1) (*a* + 1) = ± 1

If log (*a* − 1) (*a* + 1) = 1,

⇒ *a* + 1 = *a* − 1, which is never possible.

If log (*a* − 1) (*a* + 1) = − 1

*a* + 1 = (*a* − 1)− 1

*(a* + 1) (*a* − 1) = 1

*a* = ± 

But if *a* = −, is not possible as logarithm of negative numbers is not defined.

∴ *a* has only one possible value *i.e*. .

36. (a) : *X* = 897324*A*64*B*

For *X* to be divisible by 8, last three digits should be divisible by 8.

So, B can be 0 or 8.

And for *X* divisible by 9, sum of digits should be divisible by 9.

Sum of digits = 43 + *A* + *B*

If *B* = 0 then *A* will be 2.

If *B* = 8 then *A* will be 3.

So, *A* + *B* can be 2(2 + 0) or 11 (3 + 8).

37. (d) : To be divisible by 11, (*p + q* + 3) – 9 = 0 or multiple of 11.

*p + q* – 6 = 0 or 11 (if 22; *p + q* > 18 not possible)

∴ *p + q* = 6 or *p + q* = 17

For divisibility by 3, of *p + q* should be multiple of 3

⇒ *p + q* = 0 or 3 or 6 .... or 15 (*p + q ≠* 18) as in that case both should be 9, but *p > q* so *p + q* *≠* 18

So, *p + q* = 6. Satisfies both conditions. *p > q*

∴ *p* = 5 and *q* = 1 or *p* = 4 and *q* = 2

38. (d) : *abcabc = abc ×* 1000 *+ abc*

*= abc* (1000 + 1) = *abc ×* 1001

So, any number of the form “*abcabc*” must be divisible by 1001 or its factors *i.e*. 1001 = 7 × 13 × 11

Hence, all the options are correct.

39. (c) : The given number is even & should be divisible by 111. So, it would be definitely divisible by 222.

40. (c) : The value should be divisible by both 9 and 4.

sum of the digits = 11+ 2*A* (should be divisible by 9)

So *A* is 8.

Now 851580 is divisible by 4.

41. (a) : Clearly the two quantities are both integers, so we check the prime factorization of 2005 = 5 401. It can be seen that (*P, Q, R*) = (4, 0, 1) satisfies the relation.

Hence, option (a) is the answer.

42. (c) : (23)*a* + 3*a* = 8*a* + 3*a*.

This expression is always divisible by 11 (8 + 3) when *a* is odd.

43. (b) : 34*n* − 43*n*

= (34)*n* − (43)*n*

= 81*n* −64*n*

So, it should be divisible by 81 − 64 *i.e*. 17 of *n* is any positive integer.

**Alternate** **Method:**

Put value of *n* and check.

Substitute *n* = 1.

44. (a) : 4*n* + 1 represents an odd power of 4 (and hence would end in 4). Similarly, 42*n* represents an even power of 4 and would end in 6. Thus, the least number ‘*x*’ that would make both 4*n*+ 1 + *x* and 4*2n* – *x* divisible by 5 would be 1.

45. (c) : S = 20 × 21 × 22 × 23 × 24 × ....... × 30

When 2 is multiplied by 5 it gives zero.

In the above multiplication there are six 5’s. So there are 6 zeros at the end of the product. Therefore, the maximum value of *a* = 6.

46. (c) : If *A* is the sum of all the alternate digits of the number starting from the units place and *B* is the sum of all the alternate digits of the number starting from tens place, then 11’s remainder of the number *A* – *B*.

For one group of ten digits of the number,

*A* – *B* = – 5

For the 50 groups, it is – 250.

So when this number is divided by 11, the remainder will be − 8 or + 3.

Therefore 8 has to be added to the number, so that sum is divisible by 11.

47. (d) : Value for *n* = 2;

28 – 22 (15) = 256 – 60 = 196 (divisible by 2, 4, and 7) for *n* = 3;

Value = 212 − 23 (22) = 3920 (divisible by 2, 4 and 7)

48. (b) : Option (b) is obvious, as 2222 and 7777 both numbers are divisible by 101.

Now we know that if the sum of the remainders of two or more numbers are divisible by the given divisor then the required expression is also divisible by the divisor. The remainder when (2222)7777 is divided by 13 is 12 and the remainder when (7777)2222 is divided by 13 is 9.

Hence the given expression is not divisible by 13.

(Since (12 + 9) *≠* 13*m* for any positive integer *m*)

Again the remainder when (2222)7777 is divided by 99 is 44 and the remainder when (7777)2222 is divided by 99 is 77.

Hence the whole expression cannot be divided by 99.

(Since 44 + 77 = 121 which is not divisible by 99.)

49. (b) : For *n* = 1, 76 – 66 = (73)2 – (63)2

= (73 – 63) (73 + 63) = (343 – 216) (343 + 216)

= 127 × 559

Clearly it is divisible by 127, 13 & 559 (559 = 13 × 43).

50. (d) : The remainder when a number is divided by 16 is given by the remainder of the last 4 digits divided by 16.

(76*n* − 66*n* = (76)*n* − (66)*n*. This expression is divisible by = 76 − 66.)

Thus, the required answer would be the remainder of  which is 6.

51. (c) : For any integer, either *n* or (*n* + 1) is always even. Hence (a) is true. The number must be a factor of 3; sum of squares of ‘*n*’ natural nos. = . Hence option (c) is also true.

Now, if *n* = 78, the product would be

78(78 + 1)(78 × 2 + 1) = 237 *k*.

And for some other value of ‘*n*’ it would not be divisible by 237.

52. (d) : As the last digit is 0 so 15*n* is divisible by 5. It is divisible by 3, so the sum of the digits is divisible by 3.

There should be at least three 8’s. Hence the smallest for 15*n* is 8880.

⇒ *n* = 

53. (d) : *xq + xq* + 2 = *xq*(1 + *x*2)

∴ *x* = 2, 3, 5, 7, 8

For 5 values, the expression is divisible by 10.

54. (d) : =  (Rem = 4)

=  (Rem = 4)

=  (Rem = 4)

=  (Rem = 4) and so on.

55. (a) : *N* is an even number

*N* = 163 + 173 + 183 + 193 = 163 + 193 + 173 + 183

= 35(*x*) + 35(*y*) = 35(*x + y*)

So, *N* is divisible by 35.

Hence, *N* will be completely divisible by 70.

So, the remainder will be 0.

56. (c) : The remainder of each power of 9 when divided by 6 would be 3. Thus, for (2*n* + 1) powers of 9, the total remainder is 3(2*n* + 1) = 6*n* + 3. Hence, the remainder would be 3.

57. (a) : P256 = (P64)4

∴ P has the form 2k + 1

∴ it is odd ⇒ P64 is also odd.

(P64)4 when divided by 16 leave a remainder of 1 because the fourth power of any odd number divided by 16 leaves a remainder of 1.

58. (c) : We know that 5! or greater than 5! will be divisible by 5. so, Remainder when (1! + 2! + 3!.......1000!) is divisible by 5 equals to when (1! + 2! + 3! + 4!) is divided by 5.

So 

Hence, remainder obtained = 3

59. (d) : 

=  ........ (till 100 terms)

= 

= 

Hence, remainder = 6.

60. (c) : 1 + *a* + *a*2 + ..... + *a*127 = 

= 

From this expression it can be said that it is divisible by a64 + 1. Therefore, the maximum value possible is 64.

61. (d) : (1 × 1!) + (2 × 2!) + (3 × 3!) + ..... (10 × 10!)

= (2 − 1)1! + (3 − 1)2! + (4 − 1)3! + ..... (11 − 1)10!

= 2! − 1! + 3! − 2! + 4! − 3! + ..... 111 − 10!

= 11! − 1

Sum of the given series, *p* = 11! − 1

∴ *p* + 2 = 11! − 1 + 2 = 11! + 1. Hence the remainder is 1.

62. (c) : 1026 – 1 = (999.....26 times)

1026 – 1 is divisible by 3, 9, 11 & 99

but (1026 – 1) – 22 will be divisible only by 11 as 22 is even.

63. (d) : 41 + 2 + 3 + ..... + 1234 = 4(1234 × 1235)/2 = 4617 × 1235 = 4 odd

(which always end in 4 itself).

64. (b) : The unit digit of each pair is 4, and there are 50 such pairs which are multiplied together. Thus finally we get 6 as unit digit. As

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 4 × 92 | × | 43 × 94 | × | 45 × 96 | × ..... × | 499 × 9100 |
| ↓ |  | ↓ |  | ↓ |  | ↓ |
| 4 |  | 4 |  | 4 |  | 4 |

Again 4 × 4 × 4 × 4 ...... 4 (upto 50 times)

*i.e*., the unit digit of 450, which is 6.

[Since unit digit of 42*n* is 6 for *n* = 1, 2, 3, .... etc.]

65. (a) : The number in each bracket contains 7 in the units place. Therefore, the resultant product will have the units digit as the units digit of 7102, *i.e.* the units digit of 72 = 9.

66. (b) : Units digits of 729 × 345 × 343 is 5. So both (*a* and *b*) have to be odd.

*a* can take 5 odd values and *b* can take 5 odd values.

∴ 5 × 5 = 25 numbers of values can be taken by (*a, b*).

67. (a) : 302720 = 32720 × 102720

The last digit before 0 is the last digit of 32720 *i.e*. 1.

68. (d) : The unit digit of (1!)1! = 1

The unit digit of (2!)2! = 4

The unit digit of (3!)3! = 6

The unit digit of (4!)4! = 6

The unit digit of (5!)5! = 0

Now since we know that 5!, 6!, 7!, 8!, ..... all have their unit digits as zero.

Thus the sum of all the unit digit’s = 7

(Since 1 + 4 + 6 + 6 + 0 = 17)

69. (a) : Let the number = *xxxx* 9

Since, the last digit of 9even = *xxx*.... 1

And last digit of 9odd = *xx*..... 9

Then last digit of (*xxxx* 9)even (> 50) = *xx*..... 1

Then 17 × (*xx*...... 1) = *yy*....... 7

And (*yy*....... 7)4*n* = *zz*....... 1

So, the last digit of resulting number = 1

70. (d) : =  = 344 × 455 + 2144 – 72

⇒ last digit 1 × 4 + 6 = 0

71. (c) : If *p* = 3, then *n* = 5 (since *n* > 2)

If *p* = 10, then *n* = 5

If *p* = 17, then *n* = 5 and so on.

Hence *n* = 5

72. (b) : This is possible only when *n* = 4, 10, 16, 22, 28, 34, .... hence the remainder will be 3 when (*n* – 1) will be divided by 6.

73. (c) : Anil would place eight apples in the basket (as there are eight 1’s).

For the bananas, he would place six bananas (number of 2’s) and remove four bananas (number of 4’s) from the basket. Thus, there would be 2 bananas and 8 apples in the basket.

74. (d) : N = 843 – 723 – 123 = 843 + (– 72)3 + (– 12)3

= 3 × (84) (– 72) (– 12) = 3 × 84 × 72 × 12

[ 84 – 72 – 12 = 0]

= 3 × 3 × 22 × 7 × 23 × 32 × 22 × 3 = 27 × 35 × 7

Number of factors = (7 + 1)(5 + 1)(1 + 1)

= 8 × 6 × 2 = 96.

75. (a) : 3600 = 24 × 32 × 52

Factors that are perfect squares are 1, 22, 24, 32, 52, 2232, 2432, 2252, 2452, 3252, 223252, 243252. Hence these are 12 factors which are perfect square.

76. (c) : 630 = 2 × 32 × 5 × 7

No. of prime divisors = 4(2, 3, 5 and 7)

Even divisors = 1 × 3 × 2 × 2 = 12

Difference = 12 – 4 = 8.

77. (a) : 52920 = 23 × 33 × 5 × 72

Sum of factors of 52920

= 

= 15 × 40 × 6 × 57

= 205200

78. (d) : 10584 = 23 × 33 × 72

The odd factors of 10584 are 3070, 3170, 3270, 3370, 3071, 3171, 3271, 3371, 3072, 3172, 3272, 3372

The product of all these odd factors is = 318 × 712.

79. (c) : By going through to the options we find that the answer is (c), because, number of factors of 30 (2 × 3 × 5) = 8

And number of factors of 60 (22 × 3 × 5) = 12

Both the numbers have same number of prime factors.

80. (a) : 250 = 2 × 53

Number of co-primes = 250 × 

= 250 ×  × = 100.

81. (d) : 300 = 22 × 3 × 52

No. of co-primes to 300 = 100

= 26

82. (b) : It is given that LCM = 590 and HCF = 59

So, numbers can be assumed as 59*a* and 59*b*

We know that Product of two numbers = LCM × HCF

So, 59*a* × 59*b* = 590 × 59

hence, *ab* = 10 ⇒ Sets possible for *a* and *b* = (10, 1) and (5, 2)

From here the sets of value of *a* and *b* are:

I. 59 × 10 and 59 × 1

II. 59 × 5 and 59 × 2

83. (a) : Let the numbers be 16*a* & 16*b*, where *a* and *b* are coprime to each other.

16*a* × 16*b* = 7168

*ab* = 28

Only two pairs of *a* × *b* are possible which are coprime to each other as (1 × 28) & (4 × 7).

84. (b) : 25930800 = 24 × 33 × 52 × 72

So, if this number is divided by 3, then we will get a perfect square.

85. (c) : The L.C.M. of (6, 7, 8, 9, 10) = 2520  
Dividing 999999 by 2520, we get 2079 as remainder. Therefore the required number is 999999 – 2079 – 2 = 997918.

( The common difference for remainders is 2; 6 – 4 = 7 – 5 etc.)

86. (a) : For minimum number of classrooms maximum number of students should be there in a classroom.

This can be obtained by calculating the HCF of 60, 72 and 96 = 12

It means, every classroom should contain 12 student.

Hence, number of classrooms =  = 19

87. (c) : In other words, the largest number which gives the same remainder when it will divide 57, 129 and 177 is the answer.

Let the remainder be *r*.

So, the answer is the HCF of (57 − *x*), (129 − *x*) & (177 − *x*)

Now the HCF should be the factor of their difference

(129 − *x*) − (57 − *x*), (177 − *x*) − (129 − *x*), (177 − *x*) − (57 − *x*)

*i.e*. 72, 48 & 120

By observation we find that the HCF is 24.

88. (b) : (15, 3)! = Product of 3 consecutive natural numbers starting from 15, which is at least divisible by 3!.

Hence, H = 3!.

89. (b) : Let the rectangle has *a* and *b* tiles along its length and breadth respectively.

The number of red tiles

*W* = 2*a* + 2(*b* – 2) = 2(*a + b* – 2)

And the number of red tiles *R* = *ab* – 2 (*a + b* – 2)

Given *W = R*  ⇒ 2 (*a + b* – 2) = *ab* – 2 (*a + b* – 2)

4 (*a + b* – 2) = *ab*

⇒ *ab* – 4*a* – 4*b* = – 8

⇒ (*a* – 4) (*b* – 4) = 8

⇒ 8 × 1 = 8 or 4 × 2 = 8

⇒ *a* = 12 or 8

∴ 12 is the correct answer.

90. (d) : Let there be *k* number of benches.

5*k* + 4 = 11 (*k* – 4)

⇒ 6*k* = 48

⇒ *k* = 8

Hence 5*k* + 4 = 44

Thus there are 8 benches and 44 students.

**Alternate** **Method:**

Go through options and match the conditions given.

91. (c) : The HCF of 100 and 120 is 20.

Hence, HCF of (3100 – 1) and (3120 – 1) is (320 – 1).

92. (b) : 572 × 827 = 572 × 281 = (572 × 272) × 29

= 512 × 1072 = 512000 ..... 0

∴ There are total 75 (= 3 + 72) digits in the above expression.

93. (b) : *N* = 204 × 221 × 238 × 255 × ...... × 850

= (17 × 12) × (17 × 13) × (17 × 14) × (17 × 15) × ...... (17 × 50)

We are required to count the number of 5’s in

*N*, which will be equal to the number of zeros in *N*.

We will get 51 from the multiple of 5

= 15 × 20 × 25 × 30 × 35 × 40 × 45 × 50 = 510 × *K*

So, number of zeros = 10.

94. (b) : *N* = 18! × 19!

= 18! (1 + 19)

= 18! × 20

Now, 18! contain 3 zeros.

So, the number of zeros = 3 + 1 = 4

95. (b) : The number of zeros would be given by counting the number of 5‘s. The relevant numbers for counting the number of 5’s in the product would be given by:

55, 1010, 1515, 2020, 2525 .... and so on till 100100.

The number of 5’s in these values would be given by:

(5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 + 55 + 60 + 70 + 75 + 80 + 85 + 90 + 95 + 100) + (25 + 50 + 75 + 100)

= 1050 + 250 = 1300

96. (c) : 990 = 11 × 32 × 2 × 5.

The highest power of 990 which would divide 1090! would be the highest power of 11 available in 990.

This is given by = 99 + 9 = 108.

97. (c) : 3 × 62 + *A* × 6 + 3 + 4 × 62 + 4 × 6 + *A*

= 1 × 63 + 2 × 62 + 4 × 6 + 2

⇒ 108 + 6 *A* + 3 + 144 + 24 + *A* = 216 + 72 + 24 + 2

⇒ 279 + 7 *A* = 314 ⇒ 7 *A* = 35 ⇒ *A* = 5

98. (a) : Let the base be *n*, then since (44)*n* (11)*n =* (1034)*n*

Hence (4*n* + 4)(*n* + 1) = *n*3 + 3*n* + 4

⇒ 4*n*2 + 8*n* + 4 = *n*3 + 3*n* + 4

⇒ *n*3 − 4*n*2 − 5*n* = 0

⇒ *n* = 0, − 1, 5.

Base cannot be − 1 and 0.

Hence base is 5 and (3111)5 = 3 × 125 + 25 + 5 + 1 = (406)10

99. (d) : From the given options since 63 and 75 are multiple of 3, thus their remainder can not be 1.

Now checking the rest of the two option:

31 = (11111)2 = (1101)3 = (111)5

91 = (1011011)2 = (10101)3 = (331)5

100. (a) : (1)2 → (1)10

(2)3 → (2)10

(9)10 → (9)10

So, sum = 45

**Solutions Exercise – Difficult**

1. (d) : *a*741, *a*534 and *a*123 are divisible by 3 because 741, 534 and 123 and divisible by 3 hence they are not primes. a77 is also not prime as it will be divisible by 1 written 7 times or 11 times.

Hence I, II and III are correct but none of the option says that these three are correct hence option (d) that says all the them are correct is the right answer.

2. (d) : *P* = 6 ± 1. Hence *p*2 − 1 = (6 ± 1)2 − 1 = 36*k*2 ± 12*k* = 12*k* (3*k* ± 1) which is always divisible by 24.

3. (c) : From the given condition following is the result of operation −

Numbers *a* and *b* are erased and replaced with (*a* + *b* − 1) in this process we are reducing the sum of these two numbers by 1. Given that the operation was performed 39 times, hence total summation will be reduced by 39. After 39 such operations we will have only one number left on the board.

Initial sum of the numbers = 1 + 2 + 3 ..... + 40

= 40 ×  = 820

So, last number on the board = 820 − 39 = 781

4. (a) : This question should be solved by pattern method,Consider the 1st term only

= 

Consider the 1st two terms

= 

= 

Following the same pattern we can conclude that if *n* terms are there then final result is *n* −.

So, the answer is 2008 − 

5. (a) : (*ab*)2 = *ccb* hence the 2 digit number *ab* < 32. From observation *ab* = 30 or 32 doesn’t satisfy the condition hence *ab* < 30 or *a* = 1 or 2.

By observation, we have (21)2 = 441.

So, value of *b* is 1.

6. (c) : (10*p + q*) × (10*r + s*) – (10*q + p*) × (10*s + r*)

= (100*p* × *r* + 10*q* × *r* + 10*p* × *s* + *q* × *s*) – (100*q* × *s* + 10*q* × *r +* 10*p* × *s* + *p* × *r*)

= 99 (p × r – *q* × *s*)

Now, in order to the difference be maximum so *p* × *r* will be maximum and *q* × *s* will be minimum, thus *p* × *r* = 9 × 8 = 72

and *q* × *s* = 1 × 2 = 2

Hence, 99(*pr* – *qs*) = 99 (72 – 2) = 99 × 70 = 6930

7. (a) : There will be 9 single digit numbers using 9 digits, 90 two digit numbers using 180 digits, 900 three digit numbers using 2700 digits. Thus, when the number 999would be written, a total of 2889 digits would have been used up. Thus, we would need to look for the 25494th digit when we write all 4 digit numbers.

Since,  = 6373.5 we can conclude that the first 6373 four digit numbers would be used up for writing the first 25492 digits. The second digit of the 6374th four digit number would be the required answer. Since, the 6374th four digit number is 7373, the required digit is 3.

8. (d) : Any four digit number in which fist two digits are equal and alst two digits are also equal will be the form 11 × (100*x + y*) *i.e.*, it will be multiple of 11 like 1122, 3366, 2244,.... Now, let the required number be *xxyy*.

Since, *xxyy* is a perfect square, the only pairs of values of *x* and *y* that satisfy the above mentioned condition is *x* = 7 and *y* = 4.

Hence, 7744 is a perfect square.

9. (c) : Digits which can create confusion = 1, 6, 8, 9 (0 cannot create confusion because passwords has to be two–digit numbers).

Total two digit numbers with distinct digit = 81

Two digit numbers created by 1, 6, 8, 9 = 12

So, total numbers left = 69

But 69 and 96 won’t create confusion (it looks same upside down), so total numbers = 71

10. (d) : It is given that if *B* = 0, then *C* = 1

But *C* = 0 .....(1)

So, B = 1

It is also given that if *D* = 0, then *A* = 1

But for *C* = 0, *A* = *D*

So, *A* = *D* = 1 .....(2)

Then, (*A* + *B* + *C* + *D*) = 1 + 1 + 0 + 1 = 3

11. (b) : One of them will have birth year as 1948 and the other one will have 1898.

Solutions for 12 and 13:

If a student solved 200 questions and got everything correct he would score a total of 620 marks. By getting Easy question wrong he would lose 4 + 2 = 6 marks, while by not solving an Easy question he would lose 4 marks.

Similarly for Medium questions, Loss of marks = 4.5 (for wrong answers) and loss of marks = 3 (for not solved).

Similarly for Difficult questions, Loss of marks 3 (for wrong answers) and loss of marks = 2 (for not solved).

Since, he has got 120 marks from 100 questions solved he has to lose 500 marks (from the maximum possible total of 620) by combining to lose marks through 100 questions not solved and some questions wrong.

12. (b) : It can be seen through a little bit of trial and error with the options, that if he got 44 questions of Easy correct and 56 questions of Difficult wrong he would end up scoring 44 × 4 – 56 × 1 = 176 – 56 = 120

In such a case he would have got the maximum possible incorrect with the given score.

13. (a) : 32 × 4 + 1 × 1 = 130 (in this case he has solved 32 corrects from Easy, 1 correct from Medium and 1 incorrect from Difficult). Thus, a total of 34 attempts.

14. (d) : 254 + 22000 + 22x

254 is a square of 227.

Comparing the other terms with (a2 + 2ab + b2) will give us a = 1, b = 2x *–* 27

Comparing the powers

2x – 54 = 2 × 1945

⇒ 2x = 3890 + 54

⇒ x = 1972.

15. (d) : Consider the following relation and compare with *ab* = *bc*

24 = 42 ⇒ *a* = 2, *b* = 4, *c* = 2

⇒ *b* > *c*

Again, 42 = 24 ⇒ *a* = 4, *b* = 2, *c* = 4

⇒ *b* *<* *c*

So, we can’t say exactly whether *b* < *c* or *b* > *c*.

***Solutions for questions 16 and 17:***

16. (a) : PQ4 × R = P206

It is possible only when P = 1, Q = 3 and R = 9

So, Q = 3.

17. (d) : P + R = 1 + 9 = 10.

***Solutions for 18 to 19:***

P × Q × R = X × Y × Z = Q × A × Y

None of them can be 0 as then whole product will become 0, so possible digits must be from 1 to 9.

Now consider 5 it has no other multiple between 1 to 9 hence none of them can be 5.

On the same logic 7 is also eliminated.

Now possible digits are {1, 2, 3, 4, 6, 8, 9}

Consider P × Q × R = X × Y × Z and all six digits are distinct numbers.  
So, P × Q × R = X × Y × Z = Q × A × Y

LCM (1, 2, 3, 4, 6, 8, 9)

= 22 × 32 = 72 (product must be the LCM of the numbers)

72 = 1 × 8 × 9 = 6 × 4 × 3

18. (b) : X, Y, Z, P, Q, R ⇒ {1, 3, 4, 6, 8, 9}, hence only left digit is 2, so A = 2.

19. (d) : Here 0, 5 and 7 were eliminated hence sum of the digits eliminated = 0 + 5 + 7 = 12

***Solutions for 20 to 22:***

20. (d) : From the given condition Summation of ‘E’ and ‘L’ must give zero in the end so that ‘Y’ comes as it in the fourth row.

(E + L = 10)

The correct assignment of digits is

7 2 5

1 6

9 6 2 4

1 0 3 6 5

21. (d) : From the solution of last question.

22. (d) : From the solution of last question.

23. (d) : *abc* can be 370 or 371.

So, it is not possible to arrive at a unique answer.

Solutions for 24 to 26:

24. (a) : To find the required unit digit just add up the single unit digits of the two previous consecutive numbers successively and you will find that every 15th term of this sequence has unit digit zero. Hence the unit digit of the 75th term of this sequence is 0.

25. (b) : Every *n*th term has its unit digit 5 if *n* is divisible by 5 but is has unit digit zero (0) if *n* is divisible by 15. Since we have to find out the unit digit of 55th term. So here 55 is divisible by 5 but not by 15, hence the unit digit will be 5.

26. (c) : The unit digit of the sum of the 88th term plus 89th term will be equal to the unit digit of 90th term. Hence it will be zero.

27. (b) :

*P* = 

and *R* = 

Hence, *R* > *P*

Now *P* =  and *S* = 3333

Also 327 > 333 and 333 > 333

Hence *P* > *S*, *P* > *R* & *Q* > *R* (Since 327 > 333)

Thus the correct relation is *R* > *P* > *Q* > *S*.

28. (c) : 

Since 15*a*3, 6*a*2 and 5*a* would all be divisible by *a*, the condition for the expression to not be divisible by *a* would be if *b* is not divisible by *a.*

29. (b) : The value of *x* should be such that the left hand side after completely removing the square root signs should be an integer. For this to happen, first of all the square root of 3*x* should be an integer. Only 3 and 12 from the options satisfy this requirement.

If we try to put *x* as 12, we get the square root of 3*x* as 6. Then the next point at which we neet to remove the square root sign would be 12 + 2(6) = 24 whose square root would be an irrational number.

This leaves us with only one possible value *i.e*. 3.

Checking for this value of *x* we can see that the expression is satisfied as LHS = RHS.

30. (a) : *a*2 – *b*2 = (*a* + *b*) (*a* – *b*) = *m* × *n*

⇒ a =  and b = 

Now,

729 = 1 × 729 ⇒ a = 365, b = 364

3 × 243 ⇒ a = 123, b = 120

9 × 81 ⇒ a = 45, b = 36

27 × 27 ⇒ a = 27, b = 0

Thus we can express it in 4 ways.

**Alternate Method:**

The required answer is always equal to the number of ways in which a given product can be expressed as the product of two factors. (valid for odd number only)

31. (b) : The number should start with 1 as we need to minimize numerator and the sum of the digits should be maximized. So, the remaining two digits should be 9.

The value of *X* would be 199 and hence, the required difference is 9 – 1 = 8

32. (a) : The highest ratio would be a ratio of 100 in the numbers, 100, 200, 300, 400, 500, 600, 700, 800 and 900. Thus a total of 9 numbers.

33. (d) : Difference = (1027 – 4) – (1025 + a)

= 1025 (102 – 1) – (4 + a) = 99 × 1025 – (4 + a)

Now 99 ×1025 is divisible by 9.

So, the minimum value of *a* is 5.

34. (b) : Any digit written 6*n* times is always divisible by 10001. In the given number the remaining digit are 77777. So when 77777 is divided by 1001 the remainder is 700.

35. (c) : Using Wilson theorem 100! when divided by 101 the remainder is − 1.

So, required remainder = − 1 + 1 = 0.

36. (c) :

|  |
| --- |
| *ABC* |
| *CBA* |
| *A* + *C*/2*B*/*C* + *A* |

2*B* = *A* + *C* if the sum is a two digit number.

So, *A*, *B* & *C* are in an Arithmetic Progression.

Now 6 is one of the digit. So, the possible set of numbers can be (2, 4, 6) or (5, 6, 7) or (4, 6, 8) or (6, 7, 8) or (4, 5, 6) or (3, 6, 9) only in the case (2, 4, 6). The sum is a 3 digit number.

Numbers will be 642 & 246

Sum = 642 + 246 = 888

37. (c) : The last two digits of number in the expension of (7)4 = 01 (2401) and if the power of 7 is any multiple of 4 the last two digits will not change

*i.e*. (7)4 = 2401 ⇒ 01

(7)8 = 5764801 ⇒ 01

Since, power of 7 *i.e*. 2008 is a multiple of 4, the last two digits of (7)2008 will be 01.

38. (a) : For ten’s place digit, divide the expression by 100, but we know that 10! and greater than 10! is divisible by 100 because there are at least two 0’s in these numbers.

Now 1! + 2! + 3! ..... + 9! when divided the 100.

The remainder is 13.

So ten’s place digit = 1

39. (b) :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Thumb** | **Index** | **Middle** | **Ring** | **Little** |
| 1 | 2 | 3 | 4 | 5 |
| 9 | 8 | 7 | 6 | × |
| × | 10 | 11 | 12 | 13 |
| 17 | 16 | 15 | 14 | × |
| × | 18 | 19 | 20 | 21 |
| 25 | 24 | 23 | 22 | × |

Values coming at the thumb are 1, 9, 17 25, .....

These values when divided by 8 the remainder is 1

Now 1993 when divided by 8, the remainder is 1

So, 1994 will came at the index finger.

40. (d) : We have to write the expression in the form of *a*2 + *b*2 + 2*ab*

So, first we will find that the form 22*n* will be of *a*2 or 2*ab* from

**Case I:** 22*n* is of 2*ab* form

So,

Now 1 + 37 + 1029 = 38 + 1029 = 1067 Which is a odd value and can’t be equated to 22*n* as the power is even.

So, this case is discarded

**Case II:** 22*n* is of the *a*2 form

In this case 22058 should be of the 2ab form as it contain 274.

274 + 22058 + 22*n* = 

So, 2058 = 1 + *n* + 37

*n* = 2020

41. (b) : *N* = 420 = 22 × 3 × 5 × 7

Odd factors in *N* = 1, 3, 5, 7, 15, 21, 35, 105

Now 4*n* + 1 means remainder obtain when the number is divided by 4 is 1.

So, 4*n* + 1 numbers are 1, 5, 21, 105

42. (c) : 105 = 25 × 55

Now all the factors of 105 which will end in one zero will be those which have exactly 1 power of 2 and 1 – 5 powers of 5 and vice versa.

The factor 10(21 × 51) will be common in both the cases.

So, total factors = 5 + 5 − 1 = 9

43. (d) : Minimum = P11 → 1 prime factor

Maximum = p × q × r2 → 3 prime factor.

***Solutions for 44 and 45:***

A and B are the sum of the divisors of natural numbers P and P2

A – B = P1 – P2

It is possible only if P1 and P2 are prime numbers.

∴ A = P1 + 1; B = P2 + 1

44. (b) : Therefore the number of divisors of P1 × P2 where P1 and P2 are prime numbers is (1 + 1)(1 + 1) = 4

45. (d) : If P1 and P2 are prime numbers thenis necessarily a composite number and the sum of the divisors of  is .

***Solutions for 46 to 48:***

46. (b) : The position of any cell can be changed by the factors of that particular cell number.

Let us discuss the fate of any particular cell number as per the algorithm given:

Cell Number 45

Factors of 45 are (1, 3, 5, 9, 15, 45)

Initially – Closed

|  |  |
| --- | --- |
| After Step – 1 | Open |
| After Step – 3 | Close |
| After Step – 5 | Open |
| After Step – 7 | Open |
| After Step – 9 | Close |
| After Step – 15 | Open |
| After Step – 45 | Close |

It can be seen that for cell number 45, only those step numbers which are factors of 45 will have any impact of the position of cell number 45. These are going to be **–** Step 1, Step 3, Step 5, Step 9, Step 15, Step 45.

Beyond step 45, none of the steps will have any impact on cell number 45.

It can be concluded that the moment 1st factor is obtained (in the form of Step 1), cell will be opened.

And so on:

|  |  |
| --- | --- |
| 1st Factor | Open |
| 2nd Factor | Close |
| 3rd Factor | Open |
| 4th Factor | Close |
| 5th Factor | Open |
| 6th Factor | Close |
| And so on |  |

We can see that when 1st or 3rd or 5th factor or any odd number of factor is obtained, cell gets opened.

However when 2nd or 4th or 6th or any even number of factor is obtained, cell will get closed.

It is the case that a number is having only odd number of factors, cell would have remained opened? This is possible only if the cell number is perfect square.

Hence, cell numbers which will remain open = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. These ten cells will remain open. And ten prisoners will be released.

47. (b) : In the given options, only 64 is a perfect square number.

The option (b) is the number.

48. (c) : Option (c) is the answer, because only perfect squares have odd number of factors.

***Solutions for 49 to 52:***

49. (a) : 

= 

= 1 × 2 × 3 × ......... × 99 = (99!)

50. (a) : *R* (17) × *R* (19, 62)

= (1 × 2 × 3 × 4 × 5 × 6 × ...... × 17) × (19 × 20 × 21 × 22 × ...... × 81)

= 

51. (c) : *R* (2, 995) = 2 × 3 × 4 × .... × .... × 995 × 996 × 997

*R* (996, 1) = 996 × 997

∴ L.C.M. of *R* (2, 995) and *R* (996, 1)

= 1 × 2 × 3 × ..... × 997 = 997!

52. (b) : *R* (139, 2) = 139 × 140 × 141

*R* (141) = 1 × 2 × 3 × .... × 141

∴ H.C.F. of *R* (139, 2) and *R* (141) = 139 × 140 × 141

= 2743860

53. (b) : Consider following groups −

[1000 − 1099], [1100 − 1199], [1200 − 1299], [1300 − 1399], [1400 − 1499]. He can add 30 numbers in each group *i.e*. 150 numbers. Apart from these he can also add 1099 + 1100, 1199 + 1200, 1299 + 1300, 1399 + 1400, 1499 + 1500 & 1999 + 2000

*i.e*. 6 more numbers total numbers = 156

54. (b) : To maximize the value of expression we have to minimize the denominator of each term.

Therefore possible values are as follows:

*a* = 1, *b* = 2, *c* = 4, *d* = 8, *e* = 16

Now, let *p*

= 

= 

So, the maximum value is.

55. (a) : The highest power is 1 as 23 is a prime number.

If *P* is a prime number such that *n* < *p* < 2*n*, then the maximum power of *P* for  is 1.

56. (c) : In the 20’s the numbers are: 23 to 29

In the 30’s the number are: 32 to 39

Subsequently the numbers are 42 to 49, 52 to 59, 62 to 69, 72 to 79, 82 to 89 and 92 to 99.

A total of 63 numbers are there which satisfy the given condition.

57. (b) : *n* + (*n* – 1) + (*n* – 2) = *n*(*n* – 1)(*n* – 2)

⇒ 3*n* – 3 = *n*(*n* – 1)(*n* – 2)

⇒ 3(*n* – 1) = *n*(*n* – 1)(*n* – 2)

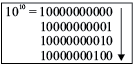
⇒ *n*(*n* – 2) = 3

⇒ *n =* 3, – 1

But the only possible values is *n* = 3

Hence, (*n*!)*n* = (3!)3 = (6)3 = 216

58. (a) :



where the sum of the digits is 2 there will be 10 values. Again the sum of the digits of 20000000000 is 2. so there are 11 values within 1010 < n < 1011.

***Solutions for 59 to 60:***

59. (a) : Number of pen > number of pencils > number of erasers

Minimum number of each of the items = 11.

So, number of erasers = 11 [If we take it 12, then minimum value of pencils = 13, and minimum value of pens = 14, sum of these three exceeds 38.]

60. (a) : Number of pens = 15, number of pencils = 12, and number of erasers = 11

Total amount = 15 × 10 + 12 × 2 + 11 × 3 = 207

61. (b) : Let the number of currency of 1 Amen, 10 Amens and 50 Amens be *a*, *b* and *c* respectively.

Then, *a* + 10*b* + 50*c* = 107.

Now, possible values of *c* = 0, 1, 2.

If *c* = 0, then *a* + 10*b* = 107. Now, number of pairs of values of *a* and *b* that satisfy the above equation are 11. These pairs :

(7, 10), (17, 9),.....(107, 0)

If *c* = 1, then *a* + 10*b* = 57. For this number of pairs of value of *a* and *b* is 6.

(7, 5), (17, 4), (27, 3), ....., (57, 0)

If *c* = 2, then *a* + 10*b* = 7. There is only one such pair of *a* and *b*, (7, 0) which satisfy the equation.

Total ways = 11 + 6 + 1 = 18

62. (b) : Let us do this iteratively. February 29th, 2012 = Tuesday

⇒ February 28th, 2012 = Monday

February 28th, 2013 = Wednesday (because 2012 is a leap year, there will be 2 odd days)

February 28th, 2014 = Thursday,

February 28th, 2015 = Friday

February 28th, 2016 = Saturday

February 29th, 2016 = Sunday.

Or, February 29th to February 29th after 4 years, we have 5 odd days.

So, every subsequent birthday would come after 5 odd days.

2016 birthday = 5 odd days

2020 birthday = 10 odd days = 3 odd days

2024 birthday = 8 odd days = 1 odd day

2028 birthday = 6 odd days

2032 birthday = 11 odd days = 4 odd days

2036 birthday = 9 odd days = 2 odd days

2040 birthday = 7 odd days = 0 odd days.

So, after 28 years he would have a birthday on Tuesday.

The next birthday on Tuesday would be in 2068 (further 28 years later), the one after that would be on 2096. His 84th birthday would again be a leap year.

Now, there is a twist again, as 2100 is not a leap year.

So, he does not have a birthday in 2100. His next birthday in 2104 would be after 9 odd days since 2096, or 2 odd days since 2096, or on a Wednesday.

From now on the same pattern continues. 2108 would be 2 + 5 odd days later = 7 odd days later.

Or, 2108 February 29th would be a Tuesday.

So, there are 4 occurrences of birthday falling on Tuesday in 2040, 2068 and 2096 and 2108.

63. (b)

I. No year can have 5 Sundays in the month of May and 5 Thursday in the month of June.

A year has 5 Sundays in the month of may ⇒ it can have 5 each of Sundays, Mondays and Tuesdays, or 5 each of Saturdays, Sundays and Mondays, or 5 each of Fridays, Saturdays and Sundays. Or, the last day of the month can be a Sunday, Monday or Tuesday.

Or, the 1st of June could be Monday, Tuesday or Wednesday. If the first of June were a Wednesday, June would have 5 Wednesdays and 5 Thursdays. So, statement (I) need not be true.

II. If February 14th of a certain year is a Tuesday, May 14th of the same year cannot be a Monday.

From February 14 to March 14, there are 28 or 29 days, 0 or 1 odd day.

March 14 to April 14, there are 31 days, or 3 odd days.

April 14 to May 14, there are 30 days or 2 odd days.

So, February 14 to May 14, there are either 5 or 6 odd days.

So, if February 14 is Tuesday, May 14 can be either Monday or Sunday. So, statement (II) need not be true.

III. If a year has 53 Sundays, it can have 5 Mondays in the month of May.

Year has 53 Sundays ⇒ It is either a non-leap year that starts on Sunday, or leap year that starts on Sunday or Saturday.

Non-leap year starting on Sunday:

January 1st = Sunday, January 29th = Sunday.

February 5th is Sunday.

March 5th is Sunday, March 26th is Sunday.

April 2nd is Sunday.

April 30th is Sunday, May 1st is Monday.

May will have 5 Mondays.

So, statement (III) can be true.

Only one of the three statements needs to be true.

64. (b) : Shopkeeper should return Rs. 1150 – Rs. 1143 = Rs. 7 but he returned Rs. 5.

Hence, it means the base used to write the number should less than 2 than the base 10.

So, base should be 8.

65. (b) : (*p*)10 ∆ (*q*)10 = (2*p* + *q* – 2)10

(101)2 ∆ (100)2 = (5)10 ∆ (4)10

= (10 + 4 – 2)10 = (12)10 = (1100)2

66. (d) : *f*(*a, b, c*) = *a + b – c*

⇒ *f*(15)8, (15)10, (15)16 = *f*((13)10, (15)10, (21)10)

= (13 + 15 *–* 21)10 = (7)10

Also (7)16 = (7)8 = (7)10